

# Iterative Rekonstruktion

Vorlesung FH-Hagenberg  
SE:MED

Computergestützte Diagnose & Monitoring

Werner Backfrieder

## Principles of Emission Tomography

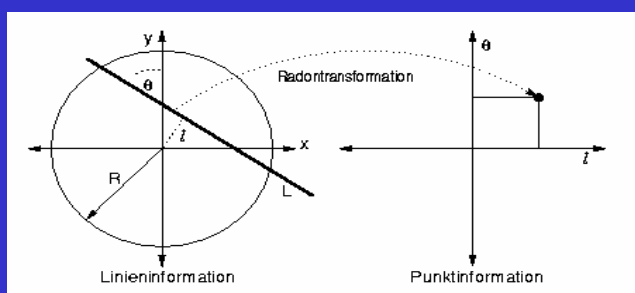
- Biological processes are visualized using radioactive tracers
- Quasi-stationary accumulation of tracer is employed for diagnostic routines (tumour staging, stroke, heart diseases)
- spatial detection of  $\gamma$ -quanta associated the location of the emission to a line of response (collimator-SPECT, coincidence measurement -PET)
- Transversal slices are reconstructed from projection data
- Stacked slices form an image volume

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## Radon Transform - Projection Geometry

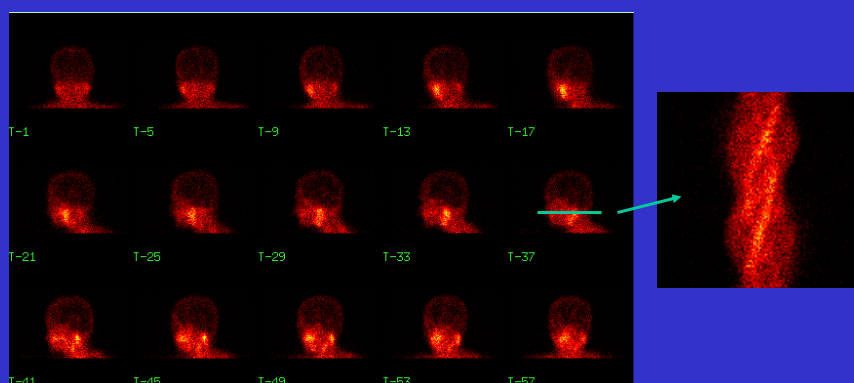
Projection or data acquisition. Information about the object is integrated along the line  $L$  and transformed into a point-information according to its co-ordinates  $r$  and  $\theta$ .



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## 3D Projection Data - Sinograms

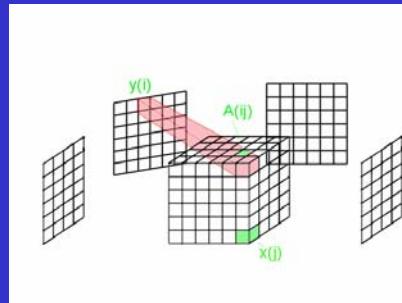


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## Iterative Reconstruction: Problem Statement

- Image volume and all projection data form the image vector  $X$  and the measurement vector  $Y$ .
- System of Equations



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## Iterative Reconstruction: System Matrix

- The elements  $a_{ij}$  of the system matrix  $A$  give the probability of the detection-event in bin  $y_i$  of a  $g$ -quantum emitted from voxel  $x_j$ .
- The following system properties are modelled by matrix  $A$ 
  - scanner geometry
  - scatter
  - attenuation
  - detector efficiency
- $Y=AX$  has no exact solution
  - noise  $\rightarrow$  inconsistent system of equations
  - number of equations  $\neq$  number of unknowns
- **Feasible solution by iterative algorithm**

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## Iterative Reconstruction: Algorithm

1. Starting image  $X_0$
2. Pseudo projection  $Y' = AX_0$
3. Comparison of measured data  $Y$  with pseudo projection data  $Y'$
4. Update of Image data  $X_n \rightarrow X_{n+1}$
5.  $X_n$  and  $X_{n+1}$  meet stopping criterium
  1.  $X_{n+1}$  is **feasible solution**
  6. else
    1. continue with **step 2**

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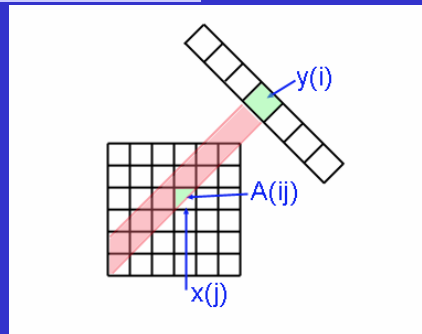
## Arithmetic Reconstruction Technique (ART)

Additive ART

$$x_j^{(n+1)} = x_j^{(n)} + \frac{y_i - \sum_j a_{ij} x_j^{(n)}}{\sum_j a_{ij}} a_{ij}$$

Multiplicative ART

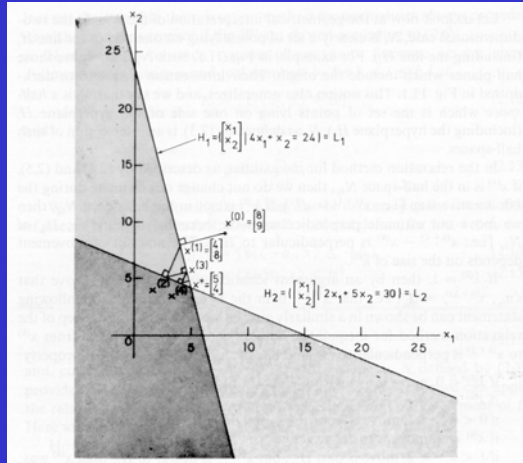
$$x_j^{(n+1)} = x_j^{(n)} * \frac{y_i}{\sum_j a_{ij} x_j^{(n)}}$$



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## Convergence of additive ART



Simple example (2 pixel and 2 projection values) for the illustration of the convergence of additive ART.

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## Statistical Models: ML-EM

Poisson distribution: 
$$p_{\lambda}(y) = \frac{\exp(-\lambda) * \lambda^y}{y!}$$

Expectation value: 
$$\lambda_i = \sum_j a_{ij} x_j$$

Measurement value: 
$$y_i$$

Log-Likelihood: 
$$p(y | x) = \sum_i (y_i \log(\sum_j a_{ij} x_j) - \sum_j a_{ij} x_j)$$

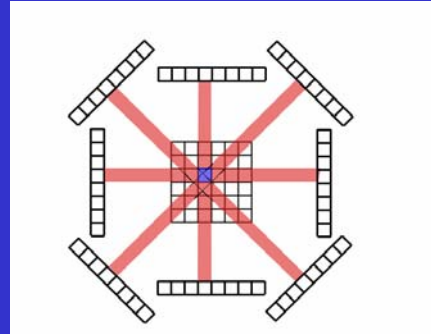
Iteration step: 
$$x_j^{(n+1)} = x_j^{(n)} * \frac{\sum_i a_{ij} \frac{y_i}{\sum_{j'} a_{ij'} x_j^{(n)}}}{\sum_i a_{ij}}$$

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# ML-EM

$$x_j^{(n+1)} = x_j^{(n)} * \frac{\sum_i a_{ij} \frac{y_i}{\sum_{j'} a_{ij'} x_{j'}^{(n)}}}{\sum_i a_{ij}}$$

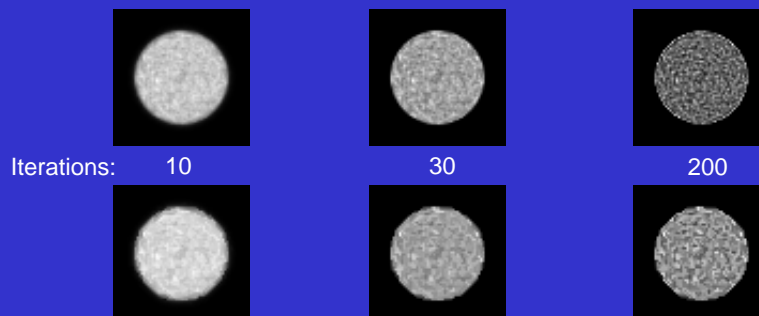


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## Convergence of ML-EM

ML-EM is a maximum entropy approach, towards high iterations noise deteriorates the image.



Prior information is implemented to provide inherent smoothing during reconstruction.

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## Bayesian Methods: Penalty Terms

MAP-density

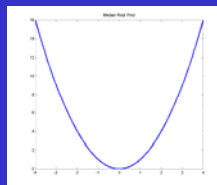
$$p(X|Y) \propto p(Y|X) * p(x)$$

Penalty term

$$P(X) = e^{-\alpha E(X)}$$

Median root prior

$$E(X) = \frac{(X-M)^2}{2M}$$



### One step late algorithm

OSL

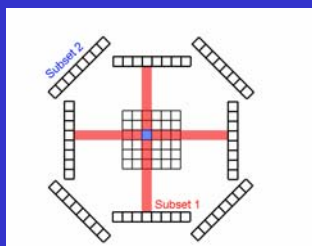
$$x_l^{(n+1)} = x_l^{(n)} \frac{\sum_k a_{kl} \frac{y_k}{\sum_m a_{km} x_m^{(n)}}}{\sum_k a_{kl} - \frac{\partial \ln P(X)}{\partial x_l} \Big|_{x_l = x_l^{(n)}}}$$

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## Acceleration: Ordered Subsets

- A series of subsets {S} of the measurement values Y is defined
- ML-EM algorithm is calculated with each subset {S} until stopping criterion is fulfilled
- Convergence is accelerated



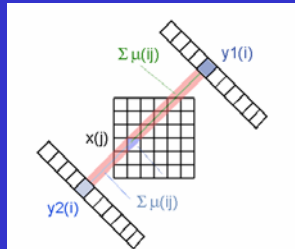
$$x_j^{(n+1)} = x_j^{(n)} * \frac{\sum_{i \in \{S\}} a_{ij} \frac{y_i}{\sum_j a_{ij} x_j^{(n)}}}{\sum_i a_{ij}}$$

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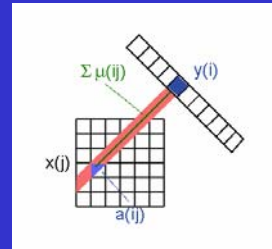
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## Attenuation Correction

PET



SPECT



$$y_{ij} \propto a_{ij} x_j \exp\left(-\int_0^x \mu(L) dL\right) * a_{ij} x_j \exp\left(-\int_x^{L_{end}} \mu(L) dL\right) =$$

$$= a_{ij} x_j \exp\left(-\int_0^{L_{end}} \mu(L) dL\right)$$

$$y_{ij} \propto a_{ij} x_j \exp\left(-\int_0^x \mu(L) dL\right)$$

$$y_{ij} \propto a_{ij} x_j \exp\left(\sum_{k \in L} \mu_{ik}\right)$$

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## Mathematical phantom

Mathematical phantom consisting of elliptical structures for simulating both distribution of radio-tracer and non-uniform attenuation:

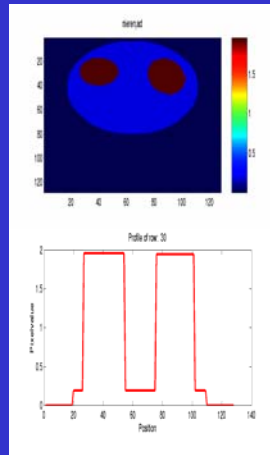
- elliptical background (big ellipsis)
- two elliptical hot spots
- uniform attenuation in big ellipsis
- high attenuation in circular area between hot spots (colon)
- Poisson distributed noise in simulated projection values

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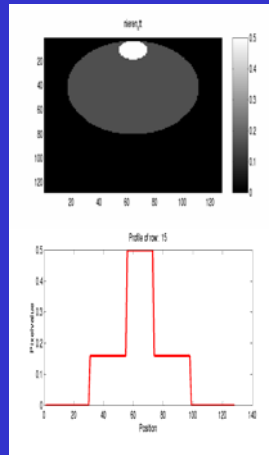
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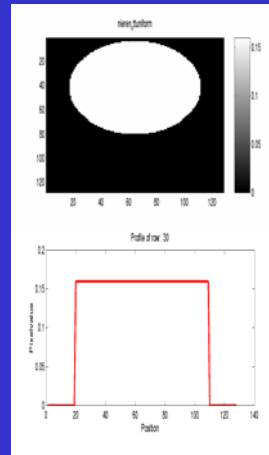
## Mathematical phantom



Distribution of radio-active tracer.



Attenuation coefficients used in simulation and non-uniform attenuation correction

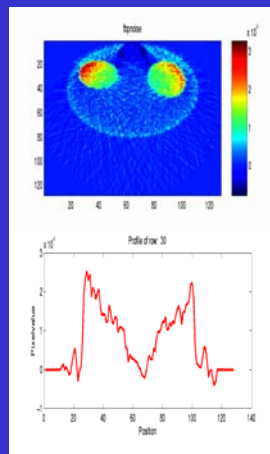


Attenuation coefficients used for uniform correction.

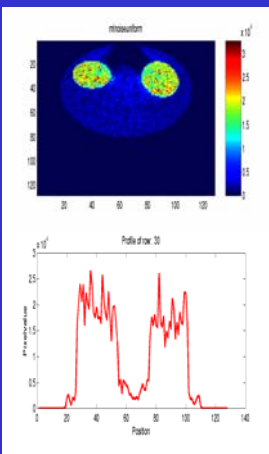
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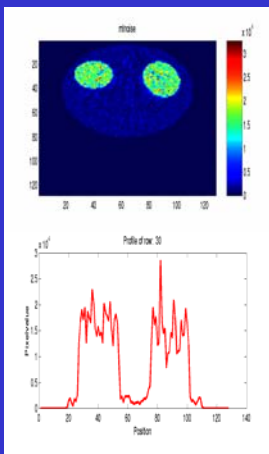
## Mathematical phantom: results 1



FBP,  $\alpha=0.8$



ML-EM, uniform, 50it

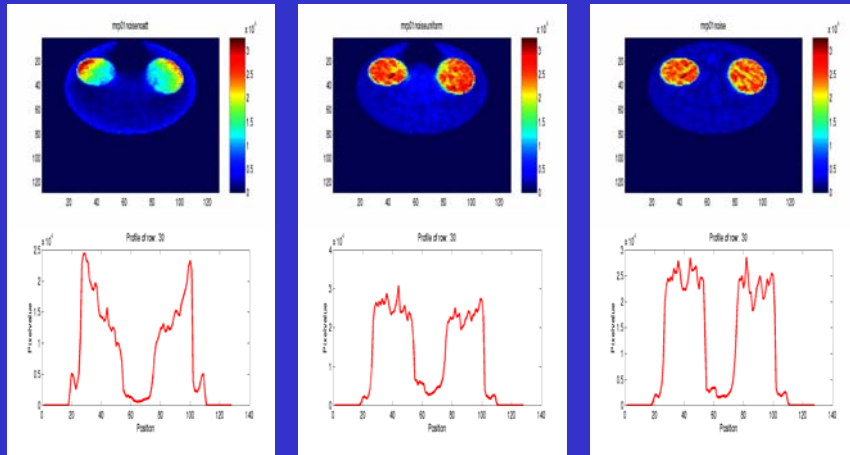


ML-EM, non-uniform, 50it

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**Mathematical phantom: median root prior, 50 iterations**



No correction

Uniform correction

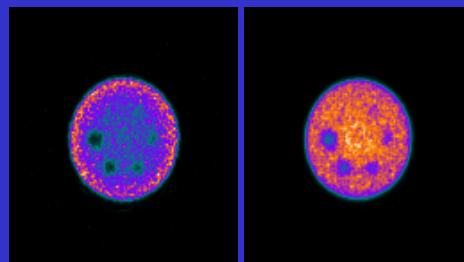
Non-uniform correction

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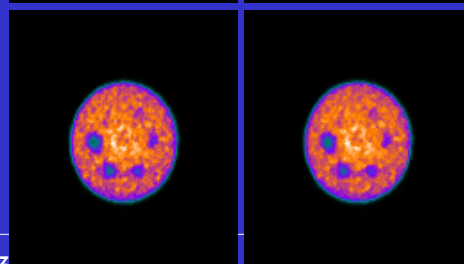
**Jaszczak phantom: 2GBq 99mTc, Picker Prism3000, FOV=46cm, 128x128 Matrix, 3.6mm slice thickness, 3° and 60s/step**

ML-EM, 50it.,  
no attenuation  
correction



ML-EM, 50it.,  
non-uniform  
attenuation  
correction

MAP-Gibbs'  
prior,  
 $\alpha=2$ ,  $\beta=0.01$ ,  
50it.,  
non-uniform  
attenuation  
correction



MRP,  $\alpha=0.1$ ,  
50it.,  
non-uniform  
attenuation  
correction

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## Alderson phantom

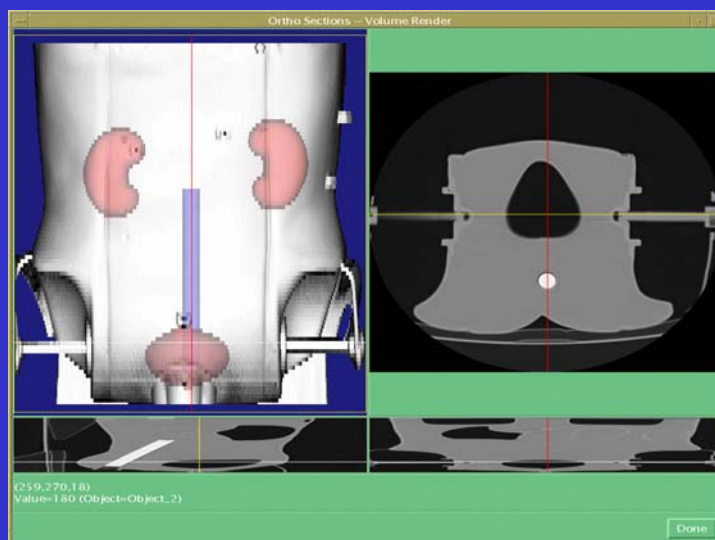
- kidneys 50 MBq  $^{99m}\text{Tc}$ , each
- bladder 150 MBq  $^{99m}\text{Tc}$
- body-background 80 MBq  $^{99m}\text{Tc}$
- colon Teflon cylinder 23mm diameter

Picker PRISM3000, 3°/step, 20s/step, circular orbit, 46cm FOV, 3.6mm slice thickness, 128x128 matrix

Philips Tomoscan SR7000, 120kV, 200mA, 34.8cm FOV, 5mm slice thickness, 512x512 matrix, planar-mode

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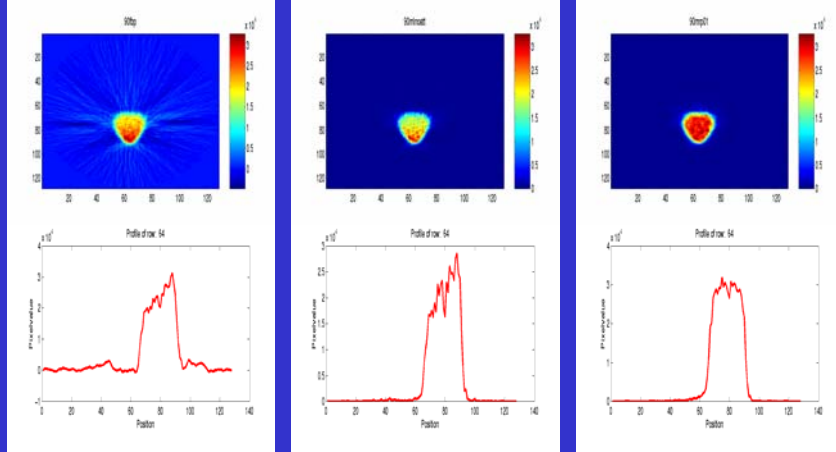


3D rendering and orthogonal sections of the Alderson phantom. Kidneys, bladder and colon are segmented.

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## Alderson phantom: results1



FBP,  $\alpha=0.8$

ML-EM, no correction

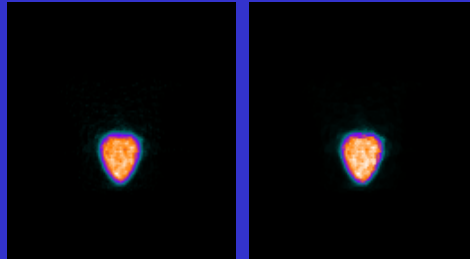
MRP, non-uniform  
correction

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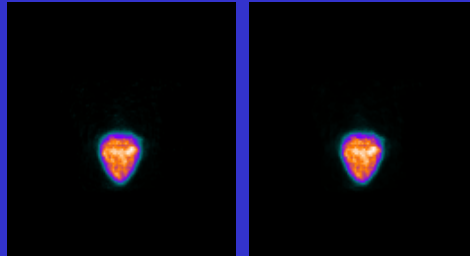
## Alderson phantom: ordered subsets (OS)

MAP-Gibbs' prior,  
 $\alpha=2$ ,  $\beta=0.01$ ,  
50it.,  
non-uniform  
attenuation  
correction



MRP,  $\alpha=0.1$ ,  
50it.,  
non-uniform  
attenuation  
correction

MAP-Gibbs' prior,  
15 subsets,  
3 it. / subset



MRP,  
15 subsets,  
3it. / subset

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## In-vivo data from head and neck area

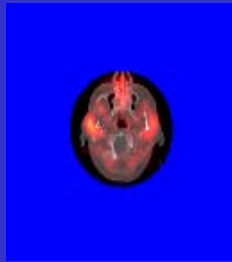
CT and SPECT data from the head and neck area:

**SPECT:** Prism3000, 99mTc Sestamibi, 128x128, FOV 46cm, 3o, 20s

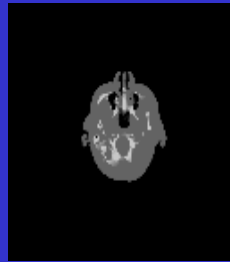
**CT:** Tomoscan SR7000, 120kV, 400mA, 512x512, FOV 185mm

Segmentation of 3 components:

bone 0.295/cm soft-tissue: 0.1596/cm air:0/cm



Overlay of CT and SPECT image data



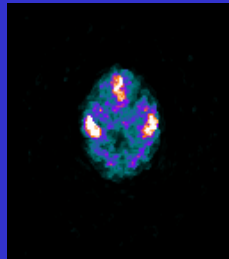
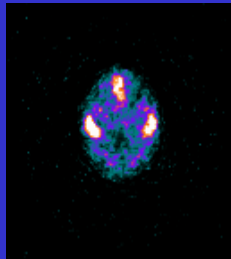
Reformatted and segmented CT image

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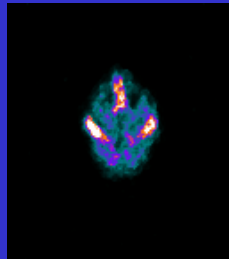
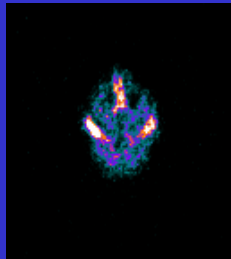
## Results: in-vivo data

MAP-Gibbs' prior,  
 $\alpha=2$ ,  $\beta=0.025$ ,  
50it.,  
no attenuation correction



MRP,  $\alpha=0.1$ ,  
50it.,  
no attenuation correction

MAP-Gibbs' prior,  
 $\alpha=2$ ,  $\beta=0.025$ ,  
50it.,  
non-uniform attenuation correction

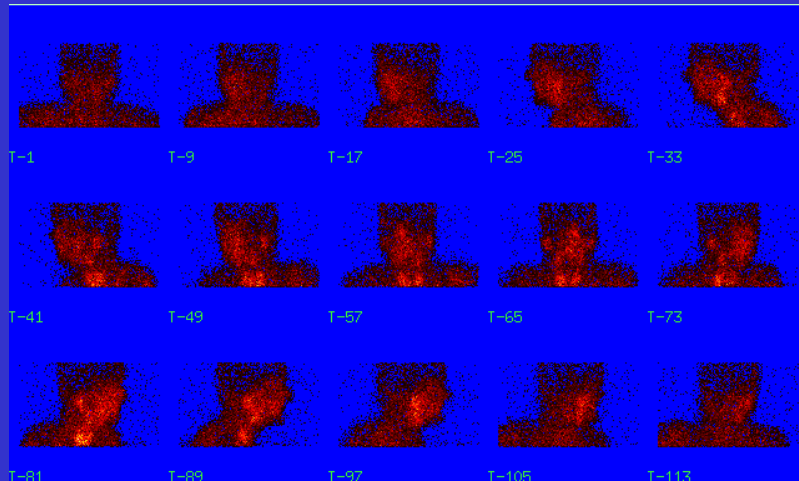


MRP,  $\alpha=0.1$ ,  
50it.,  
non-uniform attenuation correction

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## Scatter: a problem in 3D



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## Parallel Computing Concepts

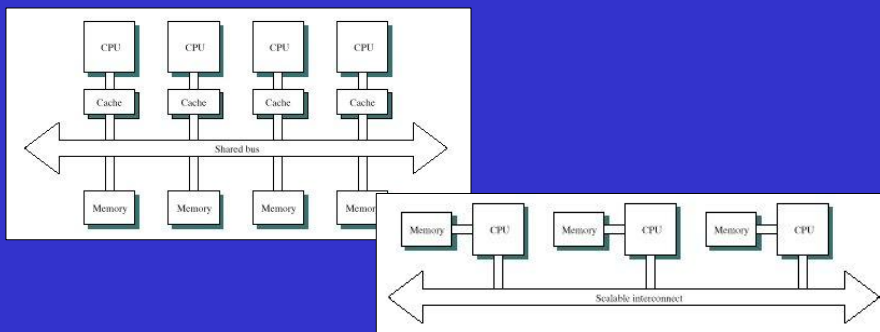
- Single task computing systems (MS-DOS)
- Multitasking OS (Unix, NT)
- Multitasking OS with multiple CPU's
- Threads on multitasking systems
- Sending messages on multitasking OS

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## Concepts of Parallel Computing

- SMP: symmetric multiprocessing
  - multiple processors use shared memory
  - bus based communication
  - distributed shared memory

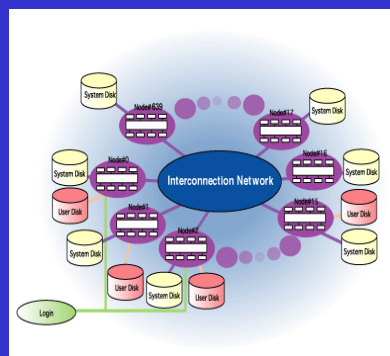


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## Concepts of Parallel Computing

- Distributed memory machines.
  - Each node has its own memory.
  - Communication via message passing
  - connected by fast Ethernet.
  - Beowulf - Linux Cluster  
zB. Avalon Cluster at Los Alamos 70 Alpha Workstations (533 MHz CPUs) 73 Gflops  
150.000\$



Earth Simulator (NEC)

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